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NÝSA, THE NKU JOURNAL OF STUDENT RESEARCH

Nýsa publishes research from students at NKU and across the commonwealth. It is published by NKU's Institute for Student Research and Creative Activity. All submissions are peer-reviewed by NKU faculty and students.

About The Title

Names are tricky things. Journals of student research are relatively common, and in looking for a name, it was important to find something evocative of the intellectual effort and exhilaration that accompany any research endeavor. If it could relate to our identity as The Norse, all the better. "Nýsa" worked perfectly. In the words of David Kime, Advising Coordinator for NKU's Honors College, who suggested it:

"The Viking raids were only one aspect of Norse society. The Norse were shipbuilders, farmers, philosophers, poets, artists, and merchants. The Norse were explorers who engineered new shipbuilding technology and navigation techniques. They sought new knowledge in the stars and from distant lands and cultures. In Old Norse, "nýsa" is a verb meaning to search or investigate; to peer into the unknown. The idea of "nýsa" applies to today's NKU students as much as it did to the Norse a thousand years ago as they peer into the unknown and produce new and exciting examples of research, scholarship, and creativity."

About The Cover

The cover and interior for this issue of Nýsa, The NKU Journal of Student Research was designed by Ivonne Ruiz, a third year student pursing a BFA in Visual Communication Design. Ivonne said about the cover: "I wanted to do something that was simple but would catch the reader's eye. When looking into the meaning of the word Nýsa I learned that it meant to peer into the unknown. This led me to want to show light shining in the darkness. With this cover I was trying to convey how you have to shine a light to investigate and make new discoveries."

From The Editor

With this seventh volume of *Nýsa*, we have published twenty-nine articles, from thirty-four student authors, in nine different fields. These aren't notably round numbers, but in thinking about all the time and effort that the authors, mentors, instructors, editors, and reviewers put into getting the scholarship to the pages of Nýsa, it is indeed an impressive quantity. Each article is connected by webs of influence to many other people, and as you read these articles, spare a second or two to ponder that web and maybe follow a thread or two. It will lead somewhere surprising.

This volume, like the previous ones, would not exist without the dedicated efforts of student and faculty reviewers, mentors, my outstanding editorial team, and the staunch support of NKU's Institute for Student Research and Creative Activity, and I am immensely thankful for all of them.

Patrick M. Hare

Editorial Board

The Effects of Mental Warm-Up on Learning Efficiency

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Psychological Sciences

KEYWORDS:
mental warm-up, learning, efficiency

Brady Cline

Brady Cline graduated from Northern Kentucky University in May 2024 with a Bachelor's Degree in Psychological Science. This research was conducted under the guidance of Dr. Kalif Vaughn for the Honors College. Since graduation, Brady works in Vocational Rehabilitation at Bawac, Inc.

Abstract

Mental warm-up represents one method of cognitive preparation, usually in the form of a task or exercise, used to improve performance on a subsequent target task. Research has shown that mental warm-up can improve performance (Irion, 1949), but experiments investigating the benefits of mental warm-up on learning are limited. The present study explores how mental warm-up in the form of solving anagrams (e.g., sircross - scissors) can impact learning. Participants were divided into three levels of mental warm-up (0, 4, or 10 anagrams) before learning Inuit-English word pairs to criterion. Analyses of learning efficiency and performance suggested no benefit of mental warm-up. Limitations and implications are discussed, as well as future research directions that might yield a benefit of mental warm-up.

Introduction

Whether getting ready for the day with a cup of coffee, studying for a test, or a coach analyzing the lineup of an opposing team prior to a game, preparation is a vital part of success. In a classroom setting, this concept can be applied to describe the preparation of one's cognitive state for learning. The Yerkes-Dodson Law (Yerkes & Dodson, 1908) states that too much stimulation, or not enough, can result in a negative impact on performance. The Optimal Arousal Theory (Yerkes & Dodson, 1908) furthers this logic by suggesting a moderate level of stimulation is best. I explored this concept in the present experiment by manipulating the degree of mental warm-up (via the number of anagrams presented to participants prior to the learning task) to examine the benefits on subsequent performance. To begin, I will review the prior literature on the benefits of warming-up, specifically the benefits of physical and mental warm-up followed by the impact of fatigue on performance. Then, I will describe the current experiment and my predictions.

Magno and Mascardo (2009) examined warm-up benefits on students in a college physical education course who were learning how to perform different swimming strokes. Since the students were learning new techniques, the experimenters gave one group of swimmers the ability to complete a warm-up exercise, whereas the other group did not complete a warm-up. The warm-up exercises included sit-ups, stretching, and running both on land and in water. They found that warming-up significantly increased swimming speed. It is important to note that this experiment involved a physical warm-up, which raises the question: Do the same benefits emerge with mental warm-ups?

In addition to benefits from physical warm-up, research shows positive effects of mental warm-up. For example, Kahol et al. (2009) studied the concept of a warm-up prior to a surgical operation. The experiment focused on surgery, and the experimenters included two levels of stress: fatigued or baseline. The experimenters defined "fatigued" as those surgeons participating in the experiment after their night call. Each surgeon participated in four sessions before night call and four sessions after night call. Prior to the operations, experimenters tasked surgeons during the experimental trials with transferring small elements from one place to another to improve hand-eye coordination. The study found that the presence of this warm-up significantly improved performance on the surgical task in both fatigued and baseline conditions. Although this warm-up consisted of a physical task, it likely provided cognitive stimulation which mentally prepared them for surgery. These results suggest strong benefits of warm-up even in tense situations such as surgical settings.

Further, there is research which investigated the effects of a purely mental warm-up on performance. Irion (1949) studied the effects of a warm-up on student learning. First, the participants completed the initial learning trials consisting of learning a list of two-syllable word pairs (e.g. Brazen-Musty). Then, the experimenters divided participants into 3 groups: one with no rest and no warm-up, one with 24 hours rest and no warm-up, and finally one with 24 hours rest and a warm-up consisting of a color naming task. Finally, the participants completed the relearning trials. The study found that the warm-up prior to relearning helped participants to relearn the content faster. The experimenters also suggested that a warm-up was most beneficial when it was related to the initial learning task.

The influence of mental warm-up was shown in a similar study conducted by Schwenn and Postman (1967), which focused on the impact of warm-up on learning word pairs on a test list. The experimenters manipulated task relatedness (related versus unrelated) as well as degree of warm-up (0, 4, or 10 warm-up trials). The related task consisted of a list of two-syllable word pairs (which were different than the test list), whereas a number guessing task was used as the unrelated task. After participants completed a warm-up (or not in the case of the control group), they were given one study trial to learn the test list, followed by a final test to criterion after two-and-a-half minutes of rest. The researchers found that performance increased with additional warm-up trials in both warm-up conditions. However, the researchers concluded that having experience with the learning task (i.e. practice effects) may have more impact on performance than a warm-up task.

Researchers have also investigated whether mental warm-up helps in learning new material. Sidney and Thompson (2019) argue the positive impact of warm-up in learning, specifically through implicit analogies in mathematics. Implicit analogies occur when students make connections from previously learned material to new topics without explicit instruction. This type of connection is especially important in mathematics. For example, teachers can utilize warm-up activities to review previous math skills that are related to novel concepts. With this previous material fresh in their minds, students can make implicit connections to the new material. Given that students make the connections themselves (rather than being told the connection explicitly), they presumably gain a deeper understanding of the material. Although anecdotal, this article suggests that warm-up activities, such as review problems in mathematics, seem to help connect related topics without the need for additional instruction, which might allow for a more holistic understanding.

Other researchers have also argued for the positive benefits of mental warm-up on learning. Na (2015) had Chinese students mentally warm-up before learning English. In this situation, a warm-up is utilized to encourage the students to achieve a mindset in which they are cognitively prepared to listen and learn the English language. Although anecdotal, the instructor noted the importance of warming up prior to leading students through their English listening exercises. Ultimately, since students were cognitively prepared, warming-up seemed to increase student attentiveness and learning ability in the classroom.

In understanding why mental warm-up might benefit students, we can explore several relevant theories. According to the Optimal Arousal Theory (Yerkes & Dodson, 1908), warming up provides a moderate amount of stimulation which might improve subsequent performance. However, the degree of warm-up may be a critical factor, as Cognitive Load Theory (subsequently abbreviated CLT; Sweller, 2011) suggests that excessive cognitive stimulation at one time leads to negative effects on task performance. This occurs due to the severe limitations within our working memories, which hinders how much information we can process at any given moment (Gruszka & Nęcka, 2017). A classroom example of CLT is when students are taking notes while also following the instructor's lecture and reading PowerPoint slides. If these combine with other factors, such as the instructor going too fast or classmates talking, the students may become overwhelmed and fail to adequately process some of the material. Additionally, excessive cognitive stimulation is overwhelming, and may cause fatigue or decreased performance and motivation to complete a given task. Another contributor to overstimulation is task switching. Research has found that by alternating between multiple tasks, additional cognitive load is induced. Given that more cognitive load is added, working memory performance is negatively impacted (Liefooghe et al., 2008).

There is empirical support for the notion that overstimulation can produce additional fatigue and negative learning outcomes. McCabe et al. (2023) asked undergraduate students affected by the COVID-19 pandemic to assess their perceived mental load and fatigue levels. Consistent with CLT, McCabe et al. found that students reported higher mental fatigue after switching to Zoom classes compared to their previous experiences with in-person classes. Although transitioning to a new learning modality was presumably stressful and a challenge, the Zoom setting likely allowed more distractions than a typical classroom setting. These distractions likely increased cognitive load and depleted cognitive resources, leading to higher rates of fatigue. Relatedly, Hopstaken et al. (2015) had students engage in an n-back task (a task designed to tax working memory) and self-report their own engagement levels after each trial. Critically, as the difficulty of the task increased, participant engagement decreased. One exception to this finding emerged when a reward was introduced before the final trial, which increased task engagement (suggesting that engagement can increase if the perceived rewards outweigh the perceived costs; see also Boksem & Tops, 2008). Pertaining to the current study, warming up might produce positive benefits on subsequent performance, but only if the warm-up activity does not cause mental fatigue or residual cognitive load. For these reasons, I manipulated the degree of warm-up to observe whether a moderate amount was best.

The current experiment involves an objective manipulation of the degree of mental warm-up and its subsequent effects on learning efficiency and memory performance. In this experiment, I had participants learn a list of Inuit-English word pairs to criterion (i.e., the items had to be successfully recalled one time before being dropped from practice). Before encoding, those in the moderate and high warm-up groups received a list of anagrams (4 or 10 items, respectively; see also Schwenn and Postman, 1967) to solve whereas the no warm-up group did not solve any anagrams. The Yerkes-Dodson Law and its Optimal Arousal Theory (Yerkes & Dodson, 1908) predicts that a moderate warm-up should produce the best learning efficiency and memory performance since it provides an ideal amount of stimulation. However, if solving anagrams is too cognitively demanding or produces residual mental load which affects encoding, then CLT might predict that the group with zero anagrams will be best for learning efficiency and memory performance. Ultimately, I predict that solving the anagrams will not be too cognitively demanding, and therefore hypothesize that those engaging in a moderate warm-up will have the best learning efficiency and memory performance on the final test.

Methods

Participants

Participants were recruited from Northern Kentucky University. To achieve 80% power to detect between-factors effects in a One-Way ANOVA, G*Power 3.1.9.7 suggests a sample size of 159 [f = 0.25, and $\alpha = 0.05$, Faul et al., 2007]. Participants were excluded if they did not reach criterion ($n = 6$) were not fluent in English ($n = 1$) or admitted to writing down the word pairs ($n = 1$). Although there were numerous individuals who did not return for Session 2 ($n = 9$), these participants were included in the analyses for efficiency (which was only measured during Session 1). In total, 16 participants ($M_{age} = 27.71$ years old, age range: 18-45; 5 males, 2 females) completed the experiment.[1]

Materials

Anagrams require no special training or prior knowledge in order to solve them; therefore, these items were utilized

[1] Note. Given that demographic information was collected during Session 2, we only obtained demographic information for the seven participants that completed both sessions.

to provide a general warm-up for the participants. These anagrams were retrieved from Srinivas and Roediger (1990) and are listed in Appendix A. Foreign-language word pairs are capable of being learned within a shorter time frame compared to some other material. Rather than using more common foreign languages (e.g., French or Spanish), I sought out foreign-language items that would be relatively unknown for most college students (Inuit-English translations). The 20 Inuit-English word pairs (e.g., ari – otter) are listed in Appendix B and were retrieved from Carrier and Pashler (1992).

Procedure

To access the experiment, participants first logged on to Sona, which is a study management service that helps in participant recruitment and study implementation. Sona directed them to an external URL which contained the experiment. After providing consent, participants were randomly assigned to receive either 0, 4, or 10 anagrams (no, moderate, and high warm-up levels, respectively). The anagrams consisted of jumbled letters of varying lengths (e.g. sirscoss), and participants were instructed to rearrange the letters to form a word (e.g. scissors). During each anagram trial, participants were each given a minimum of 15 seconds and a maximum of 30 seconds to solve each anagram.

After the warm-up phase, the copy phase began. Each word pair (e.g., mesuq – juice) was displayed on the screen for a maximum of ten seconds, and participants were instructed to type in the correct target word. After the copy phase, the criterion phase began. In the criterion phase, the participants were shown the Inuit word (e.g., mesuq) and asked to provide the correct English translation (e.g., juice) within ten seconds. If the participant answered correctly, the pair was removed from the testing stack. If the participant answered incorrectly or the time ran out, they were shown the correct answer for four seconds. After they restudied the item for four seconds, the word pair was moved to the end of the testing stack to be tested again. After all items were learned to a criterion of one correct recall, the participants were directed to an instructional page which reminded them to return for the second session in two days. After two days, Sona sent a reminder email to participants, instructing them to complete the final portion of the experiment. Within this reminder email, a web link was included, which granted participants access to the final portion of the experiment. After this reminder email was sent, participants needed to complete the final portion of the experiment within 48 hours.

During the second session, participants were given a final test on each Inuit-English word pair. After the final test was taken, the participants were asked to answer demographic and survey questions related to the experiment's administration (e.g. issues with experiment) and their perceptions of the experiment (e.g. did you feel warmed-up?). After the experiment ended, participants were debriefed and thanked for their participation.

Results

Anagram Performance

As a manipulation check, I calculated the mean anagram response rate for those given anagrams to solve. Anagram response rate was calculated by taking the number of anagrams to which the participant responded divided by the total amount of anagrams provided (4 or 10). Results suggested that participants were actively involved in anagram solving, as the mean response rate was high in both the four-anagram group (M = 0.91, SD = 0.13) and ten-anagram group (M = 0.86, SD = 0.09). Anagram performance was calculated by the number of anagrams which the participant successfully solved divided by the total of anagrams provided (4 or 10). Anagrams were solved at high levels in both the four-anagram group (M = 0.53, SD = 0.47) and ten-anagram group (M = 0.56, SD = 0.33). Overall, these results suggest that participants were actively engaged in solving the anagrams.

Learning Efficiency

Mean trials to reach criterion as a function of warm-up group are plotted in Figure 1. Trials to criterion was calculated by taking the total number of learning trials required to reach a criterion of one correct answer divided by the total number of items. To determine whether mental warm-up improved learning efficiency, I conducted a One-Way ANOVA to examine the mean number of trials to reach criterion as a function of group. Results suggested no significant difference in efficiency

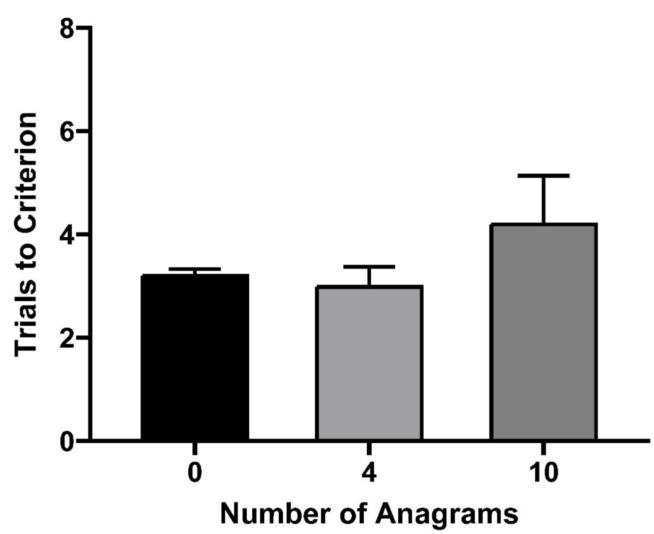

Figure 1. Mean trials to criterion as a function of group. Error bars reflect standard errors of the mean.

as a function of warm-up, $F(2, 13)=1.15$, $MSE=2.03$, $p=0.348$, $\hat{\eta}^2_G=0.150$.

Learning Performance

Final test performance as a function of warm-up group is plotted in Figure 2. Final test performance was calculated by dividing the number of items answered correctly on the final test by the total number of items. To determine whether mental warm-up improved final test performance, I conducted a One-Way ANOVA to examine final test performance as a function of group. Results suggested no significant difference in performance as a function of warm-up, $F(2, 4)=1.08$, $MSE=0.07$, $p=0.421$, $\hat{\eta}^2_G=0.351$.

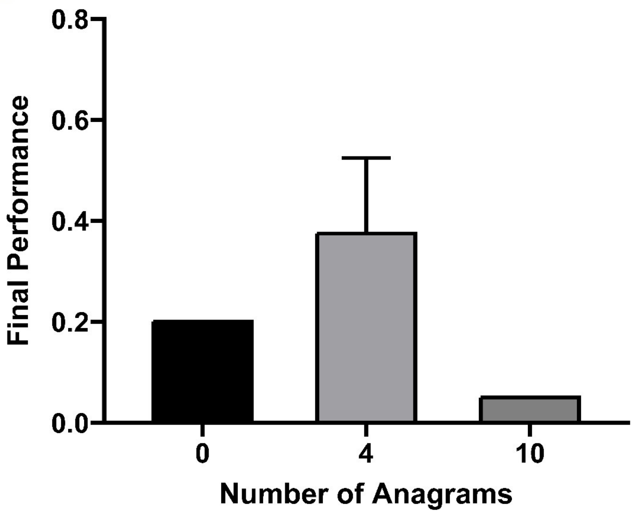

Figure 2. Mean final test performance as a function of group. Error bars reflect standard errors of the mean.

Discussion

The present study focused on exploring the impact of warm-up on learning performance. To manipulate warm-up levels (no, moderate, or high), participants were given different amounts of anagrams (0, 4, or 10, respectively) prior to learning Inuit-English word pairs. Participants were also given a final test after a 2-day delay. Analyses suggested no increase in learning efficiency or final test performance as a function of warm-up.

Although this is not consistent with the prior literature on the benefits of mental warm-up, there are a few potential reasons that might account for these findings. First, it is possible that this study did not implement a warm-up that stimulated participants enough (i.e. a warm-up that was too easy). Second, the anagrams were unrelated to the learning task. The anagrams were cognitively processed as problem solving, whereas the word pairs required associative memory. Consequently, this might have diminished or eliminated any warm-up benefits (e.g., Liefooghe et al., 2008). Third, the study had low participation and fell short of my target sample size by a considerable amount, rendering the study severely underpowered. Last, the online implementation of the study could have been a limitation, as participants might not have been as motivated to perform well or lacked complete focus on the experiment.

Given the limitations of my study, future research is warranted. First, additional efforts could be made to raise awareness of the study (i.e. flyers, recruitment emails, etc.), which might increase participation. Second, whereas this study used anagrams for warm-up, a different method might have stronger effects on performance. For instance, would different results emerge if the warm-up was more challenging, such as solving difficult math problems? Would a more targeted warm-up activity (e.g., learning word pairs in an unrelated foreign language) have produced different results? Additionally, future research could focus on the effects of warm-up for students in a physical laboratory. In-person implementation of the study might help reduce distractions and increase return rates for session two.

Ultimately, the present study investigated whether warm-up improved learning performance. As I did not find that warm-up was beneficial, these findings are inconsistent with the prior literature. Given the overall low sample size, this conclusion is highly tentative, and I hope to increase my sample size by recruiting more participants in future semesters.

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Appendix A

Word	**Anagram**
scissors	sirscoss
lemon	noeml
cloud	oulcd
snake	kanes
doorknob	knodroob
dress	sreds
spoon	ponos
pineapple	napielppe
elephant	pelethan
basket	betkas

Note. Anagrams retrieved from Srinivas and Roediger (1990).

Appendix B

Inuit-English Word Pairs

Aki	Money
Ari	Otter
Esla	Weather
Imaq	Sea
Kingu	Back
Mesuq	Juice
Nuna	Village
Nasaq	Hood
Penguq	Hill
Qalu	Ladle
Qukaq	Waist
Sanqun	Tool
Sigun	Ear
Tamlu	Chin
Teghik	Animal
Tugun	Ivory
Tuma	Path
Uneq	Underarm
Unglun	Nest
Yaquq	Wing

Note. Inuit-English word pairs retrieved from Carrier and Pashler (1992).

Does it Add Up? Comparing a Local Sample of Recreational and Prescription Drug Use to National Norms

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Faculty mentors: Heather Kissel, Ty Brumback
Psychological Sciences

KEYWORDS:
substance use trends, young adult substance use

Elizabeth G. Enneking

Elizabeth Enneking is a senior at NKU, graduating with a B.A. in psychology. She has studied for two years as a research assistant under Dr. Ty Brumback and Dr. Heather Kissel in their psychophysiology lab. During that time, she's been a part of publishing two research studies concerning alcohol and substance use in target populations. She presented her findings at NKU's 2023 Celebration of Student Research and Creativity. Elizabeth will be attending graduate school next year for Clinical Mental Health Counseling. She is looking forward to researching intersectionality and its relationship to mental health.

Abstract

Recreational alcohol and drug use in the United States is most prevalent among 18–25-year-olds (Substance Abuse and Mental Health Services Administration, 2023b). Rates of alcohol use disorder are higher in this population as well, along with high rates of illicit drug use (National Institute on Alcohol Abuse and Alcoholism, 2023; Substance Abuse and Mental Health Services Administration, 2023b). Meanwhile, usage of prescriptions drugs has only continued to increase in young adults (Martin et al., 2019). This study examined data from 2018 to 2022 to compare rates of illicit drug use and prescription medication use in a Northern Kentucky young adult sample to the rates found in the National Survey on Drug Use and Health (NSDUH). This study included 319 participants aged 18-25. We found that there were some years in which the rates of use for some substances and medications differed from the national population. However, the study sample showed no differences in use overall. Because the use rates do not differ between this local sample and the national population, emergency intervention in Northern Kentucky is not needed at this time. However, nationwide intervention may be helpful. The similarity in use rates may suggest that this sample's data on risk factors regarding alcohol use, illicit drug use, and prescription medication use may be generalizable.

Introduction

Amongst all age groups, recreational alcohol and drug use in the United States is most prevalent amongst 18–25-year-olds (Substance Abuse and Mental Health Services Administration, 2023b). The 2021 National Survey on Drug Use and Health (NSDUH) reported that the percentage of alcohol usage in the past year for 18-25-year-olds was 68.1, while it was 66.8 for all adults 18 and older (Substance Abuse and Mental Health Services Administration, 2023a). According to the National Institute on Alcohol Abuse and Alcoholism (NIAAA), alcohol use disorder is defined as "a chronic, relapsing brain disorder characterized by an impaired ability to stop or control alcohol use despite adverse social, occupational, or health consequences" (2024b). According to the 2022 NSDUH, 16.4% of 18- to 25-year-olds and 14.1% of college students of the same age range met the definition for alcohol use disorder in 2022 (National Institute on Alcohol Abuse and Alcoholism, 2024a). Meanwhile, the rate of alcohol use disorder for all adults over 18 in the United States was 11.2% (National Institute on Alcohol Abuse and Alcoholism, 2023). Also in 2022, illicit drug use among 18-25-year-olds was 40.9%, while it was 26% for adults over 18 (Substance Abuse and Mental Health Services Administration, 2023b). For the study, illicit drug use was defined as recreational use of illegal drugs or prescription medication.

This drastic increase in illicit drug use during these young adult years is likely because drinking and using illicit drugs are often viewed as a way to have more fun, to fit in, or even as a rite of passage into young adulthood, especially in college populations (Miller, 2013). In addition, the large percentage of alcohol and illicit drug usage is an issue because of the numerous biological, psychological, and social effects that illicit drug use can have on young adults, including interference with brain development, problems with the law, increased risk of alcohol use disorder later in life, and even death (National Institute on Alcohol Abuse and Alcoholism, n.d.).

Another issue becoming more relevant in conversations about young adults is the increasing rates of prescription medication usage. Prescription drug use has become more common in recent years, with antidepressants the most commonly prescribed medication category for adults and young adults (Martin et al., 2019). In the 2015 and 2016 National Health and Nutrition Examination Survey, researchers found that approximately 46.7% of the United States population aged 20 to 59 had used prescription drugs in the past 30 days (Martin et al., 2019).

For this study, we sampled students that attended a university in Northern Kentucky, as well as young adults who lived in the surrounding area. 78.6% of the university's population consists of 18–25-year-olds, which means that students on campus may be at increased risk of alcohol and recreational drug use (CollegeSimply, n.d.). Higher usage than the norm of alcohol, illicit drugs, or prescriptions could indicate a problem on campus, meaning that the students on campus could be at higher risk for medical and psychological issues.

To find out if this was the case, we wanted to investigate if the university's rates of illicit drug use and prescription drug use were statistically similar to that of the national population of young adults. So, we asked the question: How do the rates of illicit drug use and prescription drug use by 18-25-year-olds who attend university and live in the area of Northern Kentucky compare to the national rates of illicit drug and prescription drug use among 18-25-year-olds? We hypothesized that illicit drug use at the university would be lower than national averages, but that prescription medication use would be higher. We hypothesized that illicit drug use would be lower because the university is a majorly commuter campus. Therefore, on-campus parties are less likely to happen than on residential campuses, meaning that alcohol use and illicit drug use may be less likely. We hypothesized the prescription medication use would be higher because Kentucky has higher opioid prescription rates than the national average (Burnett, n.d.).

Methods

Participants

In total, we collected data for 345 participants; 26 were omitted due to incomplete or missing data. The sample participants consisted of 319 18-to-25-year-olds studying at the university or living close to the campus. 62% of the study sample was female, while 38% were male (Figure 1). 67% of the participants were White, followed by 18% Black, 5% Asian, 5% Hispanic, and 5% Multi-Racial (Figure 2).

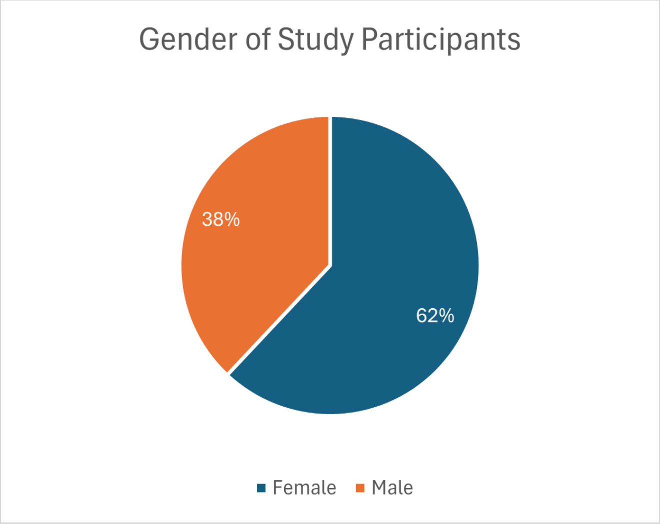

Figure 1. Gender distribution of participants.

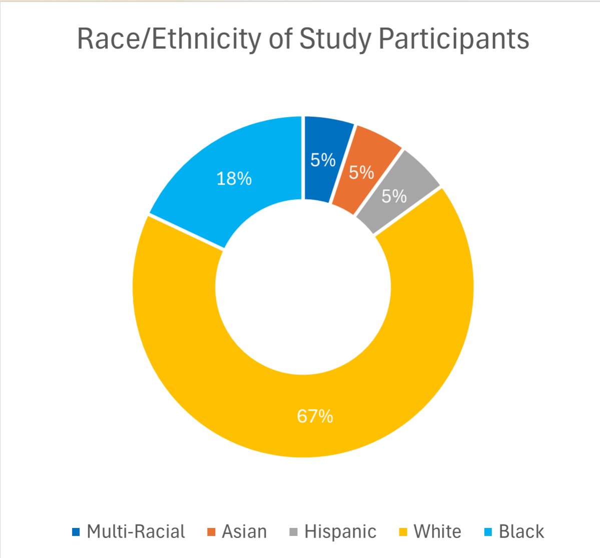

Figure 2. Race/ethnicity of study participants.

The data were collected through ongoing studies in the university's Developmental Autonomic Psychophysiology (DAP) Lab. Before participants came into the lab to participate in the study, they were screened over the phone to determine their eligibility. During the screening calls, potential participants were asked about their demographic information, including race and biological sex. Participants were recruited for the main study through posters, which were present in all buildings on campus, and Sona, a commercial platform for scheduling and managing research participation used by the university's psychology department. The participants were incentivized to participate in the main study through payments for each lab visit or for extra course credit through Sona. The participants received no compensation for the phone call.

Procedure

The current study was a cross-sectional examination comparing young adults from Northern Kentucky to national samples of young adults. The data were collected as part of ongoing studies in the DAP Lab on the university's campus. Participants were asked to complete a screening call to determine their eligibility for the in-lab studies. The call lasted 15 minutes, during which the participants answered questions about their general health, substance use, and hangover effects. During this call, potential participants were asked about their alcohol use, illicit drug use, and prescription drug use during the past month and year. Each drug they used recreationally or by prescription was recorded through Research Electronic Data Capture (REDCap), a program used for surveys and data collection. REDCap protects sample data by giving all ownership of the data over to the project owner (REDCap n.d.). Others who are granted access can input values and view data, but they do not own it. The only people with access to the main study data were those working on the project. All participants provided consent to have their data collected in an interest form that preceded the screening calls. During the screening calls, although participant demographic information was collected, names were not. All participants were assigned and referred to in the data by an identification number. The main study was approved by the university's IRB (#360 and #1296).

For this study, all screening call data on alcohol use, illicit drug use, and prescription drug use from 2018 to 2022 were compiled into one Excel spreadsheet. There were 319 participants with complete data from 2018-2022; participants were then separated by the year in which they had answered the screening questionnaires. For each year, all demographic information was added to the dataset, along with the calculated rates of alcohol and illicit drug use. Specifically, the prevalence of use of alcohol, tobacco, marijuana, barbiturates, cocaine, benzodiazepines, amphetamines, ecstasy, inhalants, and hallucinogens was calculated for the study sample by year for 2018-2021. 2022 data were not publicly available at the time of analysis. Rates of alcohol and illicit drug use in the study sample were then compared with national rates of alcohol and illicit drug use, obtained through NSDUH (Substance Abuse and Mental Health Services Administration, 2018, 2019, 2020, and 2021).

In addition to the questions about use of the substances listed above, participants were asked to give their prescription medication's name during the call. Because the screening call script inquires about the name, not the category, all provided medications needed to be investigated for their type and sorted. After classification by drug type, categories were narrowed down by sorting them into the drug's primary function. For example, fluoxetine was categorized as a selective serotonin reuptake inhibitor (SSRI), per the Food and Drug Administration (U.S. Food and Drug Administration, 2014). From there, it was classified as an antidepressant, as this is its primary function (Chu and Wadhwa, 2024).

The most common prescription drug categories for the study sample were antidepressants, insulin, attention deficit hyperactivity disorder (ADHD) medication, and birth control. The national rates of use for these four prescription medication categories were investigated using the Center of Disease Control (CDC)'s website. The CDC's rates were compared with the rates of use from the study (Table 1). The software used to analyze the data was GraphPad Prism. Fisher's exact tests, a type of Chi Squared test, were used. Fisher's exact tests are used to find an association between two types of categorical data, specifically when the sample size is small, or 20% of the cell's value less than 5 (Agresti, 1992). This type of test was used because the reported usage of most of the illicit drugs was below 5 individuals.

Results

Participants

Out of the 319 participants, 59 were surveyed in 2018, 100 in 2019, 34 in 2020, and 61 in 2021. There were fewer participants in 2020 because the university moved to virtual classes on March 26th, 2020, due to the Coronavirus pandemic (Vaidya, 2020). There were 65 participants in 2022 not included in the comparative results due to the NSDUH data for 2022 not being publicly released at the time of the analysis (May 2023). The study sample's data for 2022 is reported in Table 2.

To measure generalizability to the rest of the university's population, we conducted a series of Chi-Squared tests on the racial distribution of students in the study sample versus the reported racial distribution of the university. Through this, we found that the sample was slightly more racially diverse than the university's student population. There were fewer White people in the sample, which was statistically significant ($p < 0.001$), and more people of color, which was not statistically significant. The sample showed no significant difference in terms of gender.

Since there was no random sampling of participants across the university, and participation was instead based on interest in the study and interest in potential compensation for the main study, there was a chance of this study having low external validity. However, because the tests showed little difference between the sample and the university's population, with the only difference being fewer White participants, and because all students had access to either Sona or the QR codes on the posters, we can conclude that the study sample is representative of the university's population. The only group that the study may not generalize as well to are White students. Furthermore, the study may not generalize to the entire population of young adults in Northern Kentucky, because the university's population consists of 64% women, a large difference from the national rate of 50.9% (CollegeSimply, n.d.). There was no significant difference in terms of gender between the study sample and the university's population, and the sample has significantly more female participants than the national population. This does not interfere with the goal of the study, which is to compare the university to the national population. However, it means that the rates of alcohol use, illicit drug use, and prescription drug use reported in the study sample are likely not a good representation of the young adult population of Northern Kentucky.

Table 1. Participant alcohol use, illicit drug use, and prescription medication use data as compared to the national sample.

	2018			2019			2020			2021		
	University n = 59	NSDUH	p	University n = 100	NSDUH	p	University n = 34	NSDUH	p	University n = 61	NSDUH	p
Alcohol	66.1	73.1	0.2331	57.0	54.3	0.6738	64.7	51.5	0.1625	52.5	67.1	0.0249*
Tobacco	32.2	36.7	0.5781	40.0	35.2	0.3810	20.6	35.1	0.0986	14.8	33.7	0.0018**
Marijuana	25.4	34.8	0.1591	33.0	35.4	0.6620	20.6	34.5	0.1003	27.9	35.4	0.2694
Amphetamines	5.1	6.1	>0.9999	3.0	5.5	0.3557	0	4.6	0.3965	0	3.4	0.2553
Barbiturates	1.7	0	0.0557	4.0	0	<0.0001**	2.9	0	0.0329*	0	0	>0.9999
Hallucinogens	3.4	6.9	0.4235	1.0	7.2	0.0107*	2.9	7.3	0.5063	1.6	7.1	0.1171
Cocaine	10.2	5.8	0.1618	0	5.3	0.0116*	0	4.3	0.3945	0	3.5	0.2580
Inhalants	0	1.5	>0.9999	0	1.7	0.3910	0	1.5	>0.9999	0	1.5	>0.9999
Benzodiazepines	5.1	4.5	0.7460	0	3.8	0.0423*	0	3.3	0.6222	0	2.4	0.3928
Ecstasy	1.7	3.1	>0.9999	1.0	3.2	0.3543	0	2.5	>0.9999	0	2.1	0.6276
Antidepressants	20.3	13.2	0.1203	20.0	13.2	0.0680	20.6	13.2	0.2044	13.1	13.2	>0.9999
ADHD Medication	0	1.5	>0.9999	5.0	1.5	0.0289*	2.9	1.5	0.4166	3.3	1.5	0.2553
Insulin	6.8	2.2	0.0518	2.0	2.2	>0.9999	0	2.2	>0.9999	3.3	2.2	0.6440
Birth Control	70.0	-		32.3	-		34.8	-		32.8	-	

Notes: Data is in percentages.
Birth control usage is out of the number of participants who reported their sex as female. (2018: 20, 2019: 65, 2020: 23, 2021: 49)

Table 2. 2022 Data

	University n = 65	NSDUH
Alcohol	50.7	67.9
Tobacco	29.2	40.4
Marijuana	30.7	38.2
Amphetamines	0	3.4
Barbiturates	0	0
Hallucinogens	6.15	7.7
Cocaine	1.53	3.7
Inhalants	1.53	1.9
Benzodiazepines	0	2.1
Ecstasy	0	1.8
Antidepressants	23.1	-
ADHD Medication	6.15	-
Insulin	0	-
Birth Control	25.6	-

Notes: Data is in percentages.
Birth control usage is out of the number of participants who reported their sex as female. (39)

Alcohol Usage, Illicit Drug Usage, and Prescription Drug Usage

The results for the study overall showed few significant differences between the university and the national population. For six of the ten substances studied, there was at least one year where there was a significant difference between the study sample and the national sample. This was also true for one of the three prescription drugs studied. A detailed report of the findings can be found in Table 1.

In 2018, there were no significant differences for any of the studied substances and prescription medications.

In 2019, the study sample reported significantly less use of hallucinogens ($p = 0.0107$), cocaine ($p = 0.0116$), and benzodiazepines ($p = 0.0423$). There was significantly more usage of barbiturates ($p = < 0.0001$). However, only four out of one hundred people surveyed reported barbiturate usage. When looking at prescription drugs, we found that ADHD medication usage was significantly lower than the national sample ($p = 0.0289$).

In 2020, we again found significantly more usage of barbiturates ($p = 0.032$). However, only one person out of the sample of 32 people for that year reported barbiturate usage.

In 2021, we found significantly less use of alcohol ($p = 0.0249$) and tobacco ($p = 0.0018$).

Birth control medication usage for the study sample was recorded and investigated, however, we were unable to find a study that surveyed 18-25-year-olds on birth control medication usage in the target years. A study was found that investigated the rates of birth control usage during the target years, however, the age rage studied was 20-29, and not 18-25. When calculated, the rates for 2018 were statistically similar, but the rest of the years were statistically different, with the university's female population using significantly less birth control. However, since the age range was different, it is unknown if the study sample is statistically similar to the national population.

Discussion

The first hypothesis was that all substance use (alcohol and illicit drugs) in the study sample would be significantly less than the national average. This was not supported, as there was no significant evidence that indicated that the study sample was different than the national sample. There were a few substances where one or more years were significantly different: alcohol in 2021, tobacco in 2021, barbiturates in 2019 and 2020, hallucinogens in 2019, cocaine in 2019, benzodiazepines in 2019, and ADHD medication in 2019. However, the rest of the years for these substances were statistically similar to the national sample. No one substance was consistently significantly different from 2018 to 2021. Arria and colleagues found similar results to ours in their study on college students in the mid-Atlantic US, with marijuana use being higher, ranging from about 30-45% depending on the age of the student, and the rest of used illicit drugs being much lower, settling around the 1-10% mark (2017). Alcohol and tobacco were not included in their study.

The second hypothesis was that prescription medication use would be significantly higher than the national average. This was also not supported, as no one prescription drug was significantly different in all four years. In fact, the only drug with any significant difference was ADHD medication in 2019. The rest of the years for ADHD medication were not statistically different than the national population, and both antidepressant use and insulin use were statistically similar to national norms as well. According to Morris and colleagues, prescription medication use in college students has increased across most types of psychiatric medications, with SSRI use at 15.3% and anti-anxiety medication use at 7.6% in 2018 and 2019 (2021).

There seems to be no evidence of a concrete difference between the university and the rest of the United States in either alcohol use, illicit drug use, or prescription drug use. The evidence strongly suggests that the university's young adult population is statistically similar to the national young adult population in terms of alcohol use, illicit drug use, and prescription drug use. This suggests the possibility that there may be little cause for concern regarding drug use for this sample of students at the university, as the university does not use significantly more alcohol, illicit drugs, or prescriptions. If this is the case, then there is no need for emergency prevention, and resources can be focused on national prevention efforts instead. Nationally, 18-25-year-olds use more substances than the rest of adults 18 and older, and they are more likely to suffer from alcohol use disorder (Substance Abuse and Mental Health Services Administration, 2023b; National Institute on Alcohol Abuse and Alcoholism, 2023). The current study may not provide an accurate representation of the university's campus, due to some demographic differences and biases. However, although the demographics may differ, the similarity in use rates between the Northern Kentucky sample and national samples suggests potential overlap in risk factors. As such, the processes regarding alcohol and illicit drug use and outcomes discovered in the university sample may be useful for developing effective prevention and intervention targets across the U.S. college attending 18–25-year-old population. Going forward, this leaves a lot of room for collaboration. The study university and other universities may be able to exchange what worked well for decreasing alcohol and illicit drug use in their populations and can work together to develop a program that aids students nationwide.

Limitations

Because the study sample is statistically similar to the university in terms of gender and race, it is not representative of the national population in terms of gender and race. Overall, the findings may be more generalizable to college attending young adults, rather than 18-25-year-olds not attending college.

The main study was incentivized by compensation or extra course credit, which may mean that people who were in need of extra credit or extra cash may have been more likely to participate. To add to this, Sona is majorly used by those taking psychology classes, meaning that there may be an overrepresentation of psychology majors. Posters were placed in every building to try to even out the sample, but psychology students would have had access to the posters and would be encouraged by professors to use Sona.

Even though participants were told in the screening call that their information would not be shared with their school, their employers, or anyone else, knowing that they are reporting to an employee of their school may have caused some participants to hide their actual alcohol and illicit drug use. They even might have held back on sharing behavioral information for fear that the interviewer may judge them.

Future Opportunities

Further testing should be done with a larger sample and should continue to be done as the university's student population continues to grow and as new students arrive. Every effort to obtain a random sample should be made in order to increase external validity. Sending out an anonymous form to a randomly selected sample would address some of the limitations experienced with this study.

The university is statistically similar to the national population. Still, more resources should be focused on lowering the usage of alcohol and illicit drugs across the entire population of 18-25-year-olds to increase safety in young adults. Education about the risks associated with alcohol and illicit drug use should be increased so that students can fully know the risks before they use. Education about what alcohol poisoning and drug overdose look like, and how to intervene would be helpful as well. Resources like lectures, posters, websites, and affordable support could go a long way toward drug prevention. Implementing more kiosks that offer Narcan and other support items could help in the event of an emergency and could make students more aware of the resources available to them.

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One Small Step: Random Walks in Higher Dimensions

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Faculty mentor: Michael Waters
Mathematics

KEYWORDS:
Random walks, probability theory, computational modeling, dimensional analysis

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Abstract

If you take an infinite number of steps in random directions, what are the odds you will end up back where you started? Is the probability higher or lower if instead of taking random steps, you are flying in random directions in three-dimensional space? The answers to these questions can be summed up succinctly in the adage: "A drunk man will find his way home, but a drunk bird may get lost forever" (Durrett, 2010). That is to say, given an infinite number of steps, it is 100% probable that the random walker will return to their place of origin (or any point, for that matter) in two dimensions, but there is only about a 34% chance that same random walker will return to their place of origin if they are moving in three dimensions. The question then becomes: what happens in dimensions greater than three? While the mathematician George Pólya answered this question analytically with his theory on random walks in the early 1920's (Alexanderson, 2000), this research took a computational simulation approach using the mathematical software platform *Mathematica* to produce random walks with a large number of steps which were then processed to extract conclusions difficult to see without a modern mathematics application programming interface, such as the probability of multiple random walks within the same dimensional space intersecting each other. In this study, it was found that a conditioned-end random walk will endure according to an Inverse-Gaussian distribution. The fact that these random walks closely follow the Inverse-Gaussian distribution is significant because it suggests that their "lifetimes" can be modeled using well-established statistical tools. This opens the door to more predictive analyses of random processes in higher dimensions, with potential applications in physics, computer science, and complex systems modeling.

Introduction

The importance of studying random walks has become clear in recent years in the field of programming and game design. There have been multiple pop-culture video games such as *Stardew Valley* and *Minecraft* which implement versions of random walk programming into their games (Kobelev, 2023), both in two and three dimensions, respectively. In the natural sciences, the study of random walks directly led to the irrefutable proof of the existence of the atom in 1905, via Einstein's Theory of Brownian Motion, which is a physical analogue to three-dimensional random walks in mathematics (Chowdhury, 2008). Beyond the three-dimensional reality of everyday life, random walk theory may be useful for future physics theories such as string theory, which posits that reality may take place in as many as 26 different dimensions (Kruczenski & Lawrence, 2005).

Random walk theory has been rigorously studied, both with simulations and analytically, in the first three dimensions. While there has been substantial work on lower dimensional random walks, there remains a noticeable gap in the literature when it comes to higher dimensions—a gap this research aims to begin addressing. In order to begin research on random walks, there must first be a definition of the type and nature of the random walk being studied, as there are many different types of random walks with their own different quirks and individual rules. The type of random walks explored in this research are simple lattice random walks. In a lattice random walk, the walker may move in any of the directions defined by the dimensional space in one unit per step along an integer lattice (Montroll, 1956). This implies that, in any given step, the walker has $2 \cdot d$ 'choices' on where to take the next step, where d is the dimension. For example, a 2-dimensional random walk would be a sequence of points in the plane with integer values, where each pair of successive points is exactly one unit apart. Also, these walks are not self-avoiding, which means it is entirely possible for a walk to double back on itself repeatedly: this occurs often in the lower dimensions but decreases starkly as d increases.

Explored in this research is the idea that a random walk stops once it intersects its own path, its origin or another walk and then ends. This is analogous to a mathematically defined 'lifetime'. A 'lifetime' is often thought to have the distribution of an Inverse-Gaussian which models such non-monotonic phenomena as the time thatcertain industrial products have before breaking or needing repair (Chhikara & Folks, 1977). A random walk with a conditioned end (one which terminates when some condition is met) is also non-monotonic as its fail rate often changes very drastically within the first 25 steps. The fail rate often begins incredibly high (partially due to the fact that these walks are not self-avoiding and may double back), but then falls off quickly, especially in higher dimensions.

There are three questions explicitly considered in this research. The first question has already been answered analytically, and as such it will fulfill the important purpose of serving as a benchmark to test the simulations used in this study. All these questions use the simple lattice random walk in d dimensions (i.e., a random walk on Z^d, Z meaning 'integer'.) with a finite number of steps (n=10,000) The three questions are:

1. For a random walk on Z^d, ($1 \leq d \leq 10$), what is the probability of returning to the origin?

2. For a random walk on Z^d, ($1 \leq d \leq 10$), what is the distribution of the number of steps before intersecting itself?

3. Given two random walks on Z^d, ($1 \leq d \leq 10$) which begin one unit apart, what is the probability of intersection or crossing paths, and what is the distribution of the minimum number of steps before intersection?

The interaction between multiple walks in the same dimensional space is a topic for which less research exists. The analytical work of Pólya will be used to cross reference the result of the first question and see how close the simulated results are to the analytical results. If the simulated and analytical results agree, the second and third questions, obtained using the same base program, modified to suit the needs of the particular question, may also be valid.

Methods

A program in *Mathematica* was created which could serve as a backbone for all the other programs: a program which could take the inputs of: origin, number of steps, number of dimensions, and number of walks, and turn that into random walks with outputs that can be displayed and analyzed. This program was also modified to provide visualizations of the walks which are in two or three dimensions (Figures 1 and 2). From here, other simulations were developed to assist in answering each question individually. In Figures 1 and 2, it can be seen that the random walks presented in this study are lattice random walks with increasing dimensionality. Obviously, walks beyond the 3rd dimension cannot be shown, but simulated results can still be produced and presented for those higher dimensions.

This first program was sufficient to test against the existing Pólya's constants, as well as the ones derived from Pólya's formula in higher dimensions. This formula takes into account the number of dimensions at play, d, and gives as an output the probability that a walk will return to the origin given infinite steps, p. The general form of Pólya's formula, factoring in the

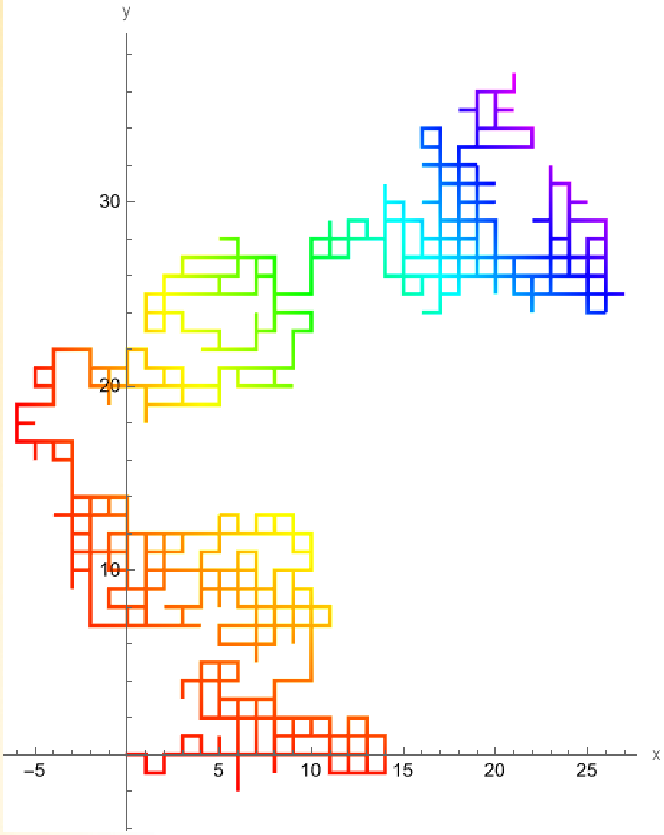

Figure 1. A lattice random walk in two dimensions. The color scale follows the number of steps from the origin (red) to violet.

angle of the next step from one point to the next in a random walk, θ, is given by

$$p = 1 - \left(\frac{1}{\pi^d} \int_{[-\pi,\pi]^d} \frac{\prod_{i=1}^{d} d\theta_i}{1 - \frac{1}{d}\sum_{i=1}^{d} \cos\theta_i} \right)^{-1}$$

The second program tested how many steps until a walk first intersected itself, and then the program terminated the walk and output the points the walk covered and how many total steps it took to terminate. This program also recorded each walk's number of steps individually, allowing the user to model the distribution of steps before self-intersection. These distributions were then tested using several theoretical distributions, giving p-values as a representation of how likely the selected distribution would suit the data by chance. The *p*-value of the Inverse-Gaussian distribution in every case tested gave a value in the range of 10^{-13} to 10^{-9}, far exceeding the required 0.05 we would expect of a valid fit's *p*-value.

The third program tested for the minimum number of steps at which two separate random walks in the same dimensional space first intersected. This program also recorded the number of non-intersecting walks. In Figure 3, the two walks start at

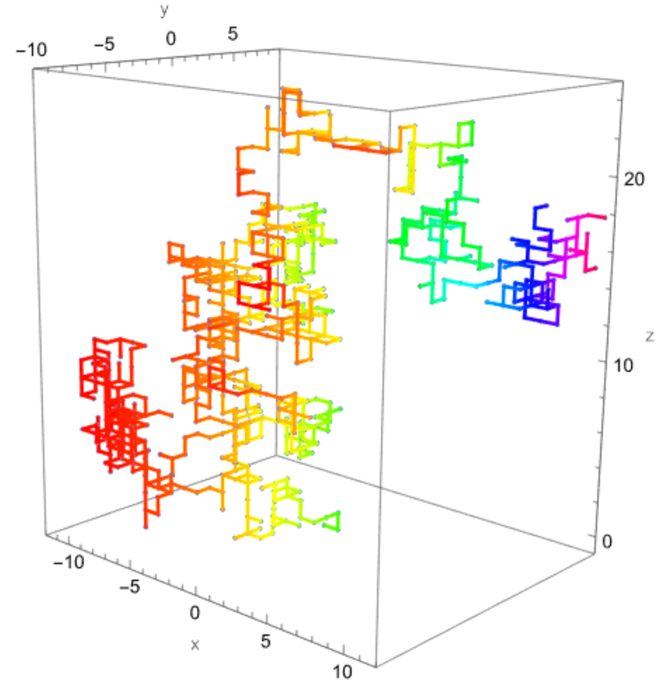

Figure 2. A lattice random walk in three dimensions. The color scale is the same as in Figure 1.

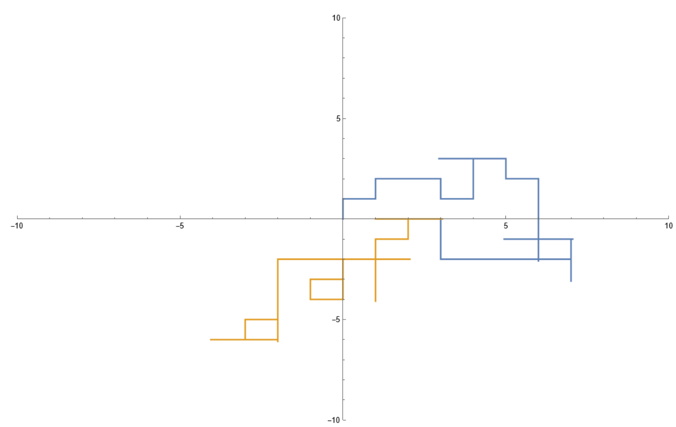

Figure 3. Two walks (blue, yellow) in two dimensions intersecting at the point (3,0).

(0,0, blue) and (1,0, yellow), and intersect after some time at the point (3,0).

Results and Discussion

By using Pólya's formula, the necessary constants to test the simulation against were calculated in dimensions one through ten. Using these values, a 99% confidence interval was found for the probability that a walk will intersect its own origin and compared to the analytical value from Pólya's formula, whose values are given in Figure 4. The simulation gave results which were found to be consistent within the margins of error for dimensions above the second, which is to be expected. A number of steps equal to 10,000 was chosen to be sufficiently

Table I. Pólya's constants for origin intersection

d	Pólya's Constant
1	100%
2	100%
3	34.1%
4	19.3%
5	13.5%
6	10.5%
7	8.58%
8	7.29%
9	6.34%
10	5.62%

large to emulate infinite steps. The simulated results paired with the 99% confidence intervals are shown in Figure 4. Pólya's constants, as calculated from Pólya's Formula, are given in Table 1 (Finch, 2003).

For Question 2, consistent success was found fitting the Inverse-Gaussian 'lifetime model' fit (Figures 5 – 8). Random walks tended to die off quickly (within 25 steps) when they terminated on self-intersection. Walks in lower dimensions tended to have shorter lifetimes and many fewer outliers (walks which last much longer before self-intersection), but most of the walks in higher dimensions tended to live longer with more outliers. It was observed that many walks of higher dimensions exhibited outliers in the hundreds of steps before self-intersection, but such outliers were nonexistent in the lower dimensions. The longest walks observed before self-intersection were in the tenth dimension, while the shortest by far were in the first, typically dying off after two steps. In higher dimension random walks which had the self-intersection condition for termination, it was found that the Inverse-Gaussian peak was severely flattened, which is to be expected. This can be seen in Figures 7 and 8, where the walks that did self-intersect were much more likely to 'wander' before doing

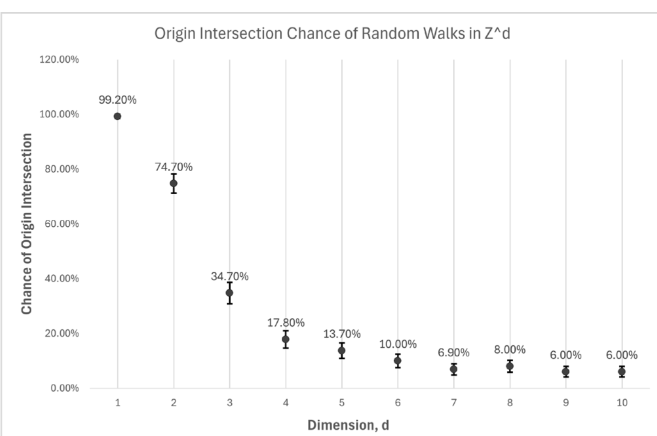

Figure 4. 99% confidence interval for origin intersection.

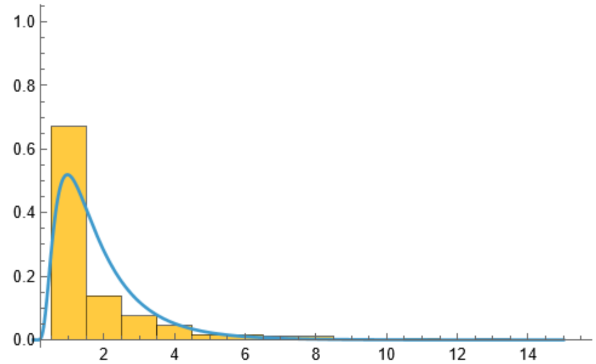

Figure 5. Fitted histogram of 2nd dimensional random walks which self-intersected.

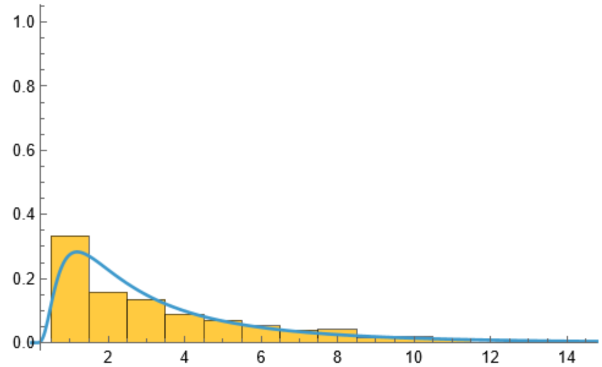

Figure 6. Fitted histogram of 3rd dimensional random walks which self-intersected.

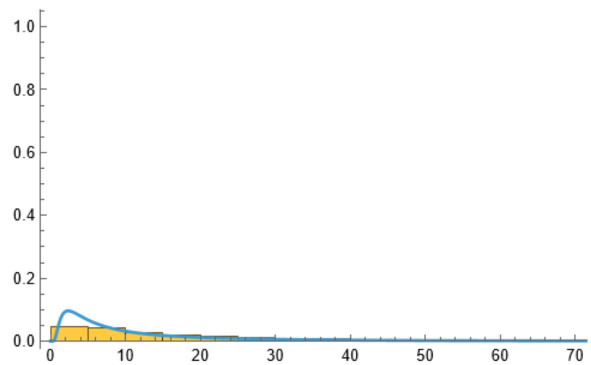

Figure 7. Fitted histogram of 9th dimensional random walks which self-intersected.

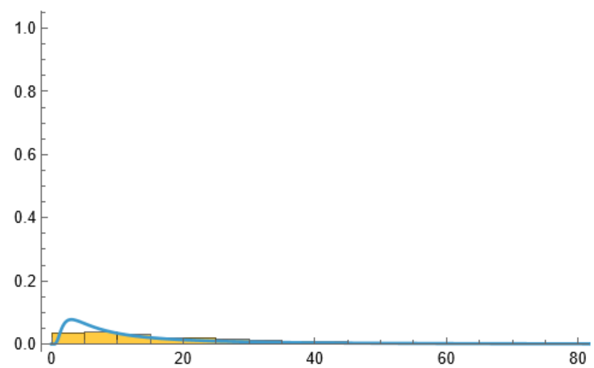

Figure 8. Fitted histogram of 10th dimensional random walks which self-intersected.

so. In the conditioned-end random walk figures (Figures 5-8; 10-13), the horizontal axis represents the number of steps until the end-condition was met, and the vertical axis represents the proportion of walks which ended on that range of steps. For example, if the first bin of the histogram is step 1 and it is at ~0.8 (as seen in figure 12), this implies that ~80% of walks reached their end-condition on step one.

After finding the distribution for an individual walk's chance of self-intersection, a second walk was introduced in the same dimensional space to see if the same trends persisted with intersecting pairs of walks. The data are shown for this simulation paired with a 99% confidence interval in Figure 9.

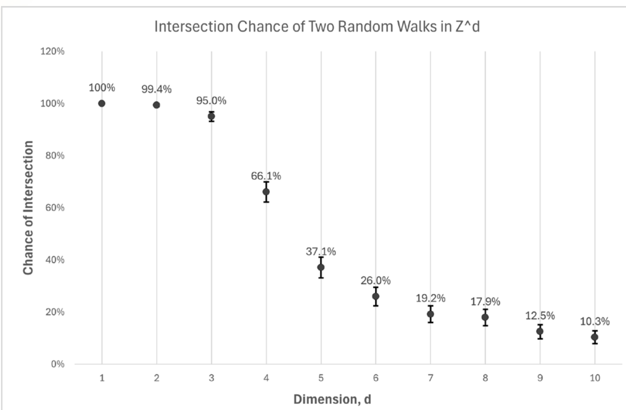

Figure 9. 99% confidence interval for intersection probability.

So, the probability that two random walks intersect is effectively certain in dimensions less than four, but then steadily falls off to the point that only one in ten walks are intersecting in dimension ten. The distribution of intersecting walks was also recorded and fitted with Inverse-Gaussian distributions (Figures 10 – 13). The walks in lower dimensions tended to have much longer lifetimes with more outliers, but the walks of higher dimensions tended to die off quickly and have very few outliers. This can be explained by the increased number of choices in the higher

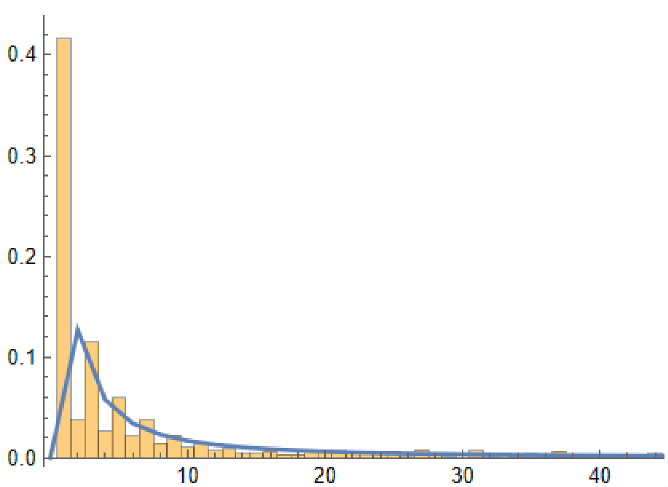

Figure 10. Fitted histogram of 2nd dimensional random walk pairs which intersected.

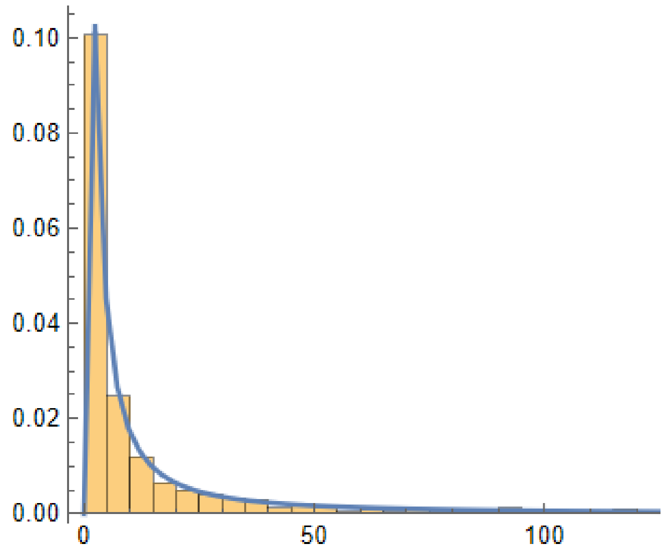

Figure 11. Fitted histogram of 3rd dimensional random walk pairs which intersected.

dimensions, lending itself to a propensity for walks to get away from each other very quickly and very early. As examples, the distribution of intersections in the second and third dimensions are shown in Figures 10 and 11. Notice that most walks that intersect do so within the first few steps, but there are still very many walks which intersect in the later parts of their lifetime.

Contrastingly, in the distributions of the ninth and tenth dimensions (Figures 12 and 13), the overwhelming majority of walks which intersect do so within the first 10 steps.

In this research, both Pólya's constants on random walks were verified using simulations and progress was made on the question of how multiple random walks interact in the same dimensional space. To a high degree of accuracy, the Inverse-Gaussian distribution suits both the number of steps before any one walk intersects itself as well as the number of steps before a walk first intersects another walk in the same dimensional space. It was also shown, to a high degree of statistical certainty, that the 'lifetime' model of conditioned-end random walks can be seen as a valid way to analyze and predict future behavior of such walks, as well as their interactions with other walks in the same dimensional space.

A limitation for this study was the amount of computing power required to produce these data. In order to increase the number of steps in each walk from 10,000 to 100,000 and the number of walks from 1,000 to 10,000 (the original goals for data collection for this study), it would require many datapoints which would take days of computing power for each run of the simulation. Statistically, the data is more than sufficient according to the Central Limit Theorem but comparing the data to the expected results according to Pólya's numbers in the 1st and 2nd dimensions was troublesome as those dimensions

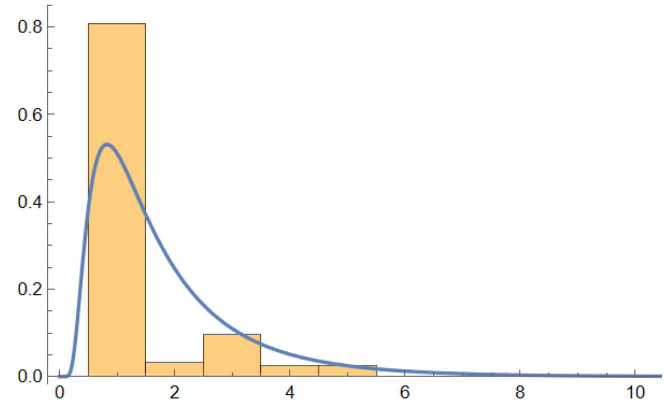

Figure 12. Fitted histogram of 9th dimensional random walk pairs which intersected.

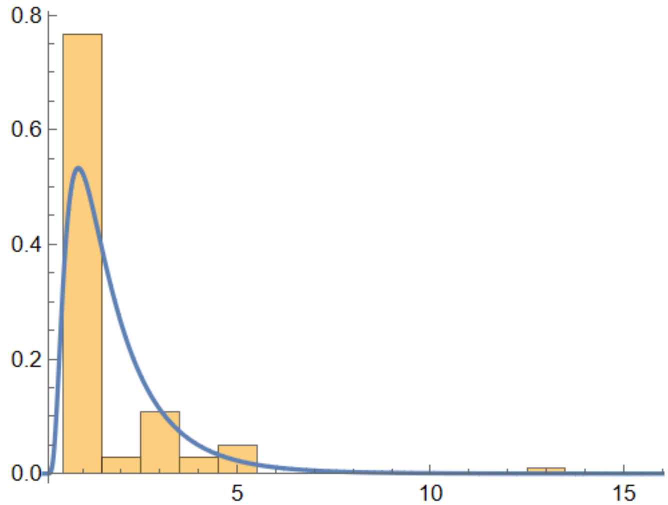

Figure 13. Fitted histogram of 10th dimensional random walk pairs which intersected.

require a much higher number of steps in order to satisfy the 'sufficiently large' condition.

Conclusion

This study successfully validated its methodology by reproducing the known results for the probability of a random walk's return to its origin (Question 1) in dimensions one through ten, as predicted by Pólya's analytical formulas. This cross-verification not only confirms the efficacy of the computer simulation framework used in this research but also establishes a robust foundation for exploring more nuanced aspects of random walk behavior.

More importantly, the research ventured into relatively uncharted territory by addressing Questions 2 and 3. The examination of the distribution for the number of steps before a walk intersects itself (Question 2) revealed that the "lifetime" of a random walk, when modeled through an Inverse-Gaussian distribution, exhibits marked differences between lower and higher dimensions. This finding enhances our understanding of random walk dynamics, particularly emphasizing that walks in higher dimensions tend to endure significantly longer before self-intersection, albeit with the presence of few outliers in the simulation.

Similarly, by introducing a second walk and analyzing the probability and distribution of the minimum number of steps before their first intersection (Question 3), the study uncovered that while intersections are virtually guaranteed in lower dimensions, the likelihood steadily diminishes as the dimensionality increases. This insight into the interplay between multiple random walkers opens promising avenues for further research on their interaction mechanisms in multi-dimensional spaces, particularly in a non-lattice framework.

Together, these results not only reaffirm the established theoretical predictions but also contribute insights into the behavior of conditioned-end random walks. The successful application of simulation techniques has thus provided a more comprehensive understanding of both individual and intersecting random walks, bridging gaps in the literature and offering a quantitative framework for future investigations in higher-dimensional probabilistic models.

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