Chapter 5

Exercise Set 5.1

Find the greatest common factor (GCF) of the numerator and the denominator of each rational expression. Write the expression by factoring out the GCF in both the numerator and the denominator. Then divide out the GCF to write the rational expression in simplified form.

\[
\begin{align*}
1. \quad & \frac{45a^3b^4}{9a^3b} \\
2. \quad & \frac{8a^{10}b^3}{6a^3b} \\
3. \quad & \frac{3x(x - 1)^2}{5x^2(x - 1)} \\
4. \quad & \frac{24x^3y^5}{30y^3z^2} \\
5. \quad & \frac{x^2(x + 3)^3}{x^3(x + 3)(2x - 1)} \\
6. \quad & \frac{15(x + 1)^2(x - 1)}{48(x + 1)^3(2x + 3)}
\end{align*}
\]

Simplify the rational expression.

\[
\begin{align*}
7. \quad & \frac{18x^4y^7}{24x^8y^4z} \\
8. \quad & \frac{6(x + 4)^3(x - 2)^2}{30(x + 4)^2} \\
9. \quad & \frac{4(a - b)(a + b)^2}{7(b - a)(a + b)^2} \\
10. \quad & \frac{(x - 2)(3x + 5)^2}{2 - x)(3x + 5)^2} \\
11. \quad & \frac{x^3 + 3x^2}{x^3 + 2x^4} \\
12. \quad & \frac{3x^2 - 15x}{12x - 60} \\
13. \quad & \frac{a^2b^2 + a^2b^4}{a^2b^2 + a^4b^2} \\
14. \quad & \frac{x^2y}{x^2y + x^4y^2} \\
15. \quad & \frac{x^3yz + xy^3z + xyz^3}{x^2y^2z^2} \\
16. \quad & \frac{6t^4 - 18t^3}{4t^2 - 12t} \\
17. \quad & \frac{t^3 - 2t^2 + t}{t^2 - t} \\
18. \quad & \frac{2a^2b^2 - 10a^6b^8}{2a^2b^2} \\
19. \quad & \frac{x^2 - 4}{x + 2} \\
20. \quad & \frac{x^2 + 4x + 3}{x + 1}
\end{align*}
\]
21. $\frac{x^2 + 6x + 8}{x^2 + 5x + 4}$

22. $\frac{(a-3)^2}{a^2 - 9}$

23. $\frac{x^2 + 2x - 3}{x^2 + x - 6}$

24. $\frac{6x + 12}{x^2 + 5x + 6}$

25. $\frac{4x^2 - 4}{12x^2 + 12x - 24}$

26. $\frac{4y^3 + 4y - 8y}{2y^3 + 4y - 6y}$

27. $\frac{y^2 - y - 12}{y^2 + 5y + 6}$

28. $\frac{2x^2 + 5x - 3}{3x^2 + 11x + 6}$

29. $\frac{x^2 + 2xy + y^2}{3x^2 + 2xy - y^2}$

30. $\frac{2x^2 + 6xy + 4y^2}{4x^2 - 4y^2}$

31. $\frac{6x + 12}{4x^2 + 6x - 4}$

32. $\frac{x^7 + 4x^6 + 3x^5}{x^4 + 3x^3 + 2x^2}$

33. $\frac{3x^2 + 12x + 12}{9x^2 - 36}$

34. $\frac{t^3 + t^2}{t^2 - 1}$

35. $\frac{t^2 - 3t - 18}{2t^2 + 5t + 3}$

36. $\frac{2t^4 - t^3 - 6t^2}{2t^2 - 7t + 6}$
Exercise Set 5.2

Perform the multiplication or division and simplify.

1. \(\frac{x^2}{4y} \cdot \frac{2x^3}{y^2}\)

2. \(\frac{y}{8} \cdot \frac{10}{y^3}\)

3. \(\frac{a^2}{a+2} \cdot \frac{6a+12}{a^2}\)

4. \(\frac{8a-6}{5a+20} \cdot \frac{2a+8}{4a-3}\)

5. \(\frac{x^3}{y} \div \frac{x^3}{y^2}\)

6. \(\frac{2a+3}{a^3} \div \frac{6a+9}{a^4}\)

7. \(\frac{x-1}{(x+3)^2} \div \frac{1-x}{(x+3)^3}\)

8. \(\frac{4x-1}{3x+2} \cdot \frac{9x^2+6x}{1-4x}\)

9. \(\frac{3x^2}{x^2-9} \cdot \frac{x+3}{12x}\)

10. \(\frac{2x^2+7x-4}{2x^2-3x+1} \cdot \frac{3-x}{x-3}\)

11. \(\frac{x^2-x-6}{x^2-1} \cdot \frac{x+1}{x-3}\)

12. \(\frac{x^2+5x+6}{x^2+2x} \cdot \frac{x^3+x}{x^2+4x+3}\)

13. \(\frac{x^2y+3xy^2}{x^2-9y^2} \cdot \frac{x^2-2xy-3y^2}{5x^2y}\)

14. \(\frac{2x^2+3x+1}{x^2+2x-15} \div \frac{x^2+6x+5}{2x^2-7x+3}\)

15. \(\frac{x^4}{x+2} \div \frac{x^3}{x^2+4x+4}\)

16. \(\frac{3x^2+2x-1}{x^2-1} \cdot \frac{x^2-2x+1}{3x^3-7x+2}\)

17. \(\frac{2a^2-ab-b^2}{a^2-2ab+b^2} \cdot \frac{2a^2+ab-3b^2}{2a^2+3ab+b^2}\)

18. \(\frac{2x^2-15}{x^2-4x-5} \cdot \frac{x^2+8x+7}{x^2+7x+12}\)
Exercise Set 5.3

Suppose the expressions given are denominators of rational expressions. Find their least common denominator (LCD).

1. $x^3 y^5, x^2 y^2 z$
2. $2a^3 b^5, 3a^6 b^2$
3. $12(x-1), 9(x-1)^3$
4. $(x+2)^3 (x+3), (x+3)^2 (x+4), (x+4)^3$
5. $x^2 + 5x + 6, (x+2)^2$
6. $15x^2 (y+1), 21x(y+1)^3$
7. $x^2 - 25, x^2 + 8x + 15$
8. $t(t^2 - 1), t^3 (t+1), t^2 (t-1)$

Write each pair of rational expressions as equivalent rational expressions with their LCD as the denominator for both.

9. $\frac{7}{2x^3}, \frac{5}{3x^2}$
10. $\frac{1}{4ab^3}, \frac{1}{a^3 b}$
11. $\frac{1}{12x^2 y^3}, \frac{1}{18x^3 y}$
12. $\frac{x+1}{4xy}, \frac{5y}{6x^2}$
13. $\frac{3}{x(x-1)}, \frac{7}{(x-1)^2}$
14. $\frac{x}{4x+4}, \frac{1}{x^2 - 1}$
15. $\frac{x}{x^2 + 4x + 3}, \frac{x + 5}{x^2 + 3x + 2}$
16. $\frac{x + 3}{x^2 - x - 2}, \frac{6x}{x^2 - 4x + 4}$

Perform the addition or subtraction and simplify. Identify the LCD in each case.

17. $\frac{y}{7x^2} - \frac{4}{7x^2}$
18. $\frac{4}{(x+1)^2} + \frac{9}{(x+1)^2}$
19. $\frac{x + 2x}{4}, \frac{3}{3}$
20. $\frac{1}{2a} + \frac{4}{3a}$
21. $\frac{1}{s} + \frac{1}{t}$
22. $\frac{2}{3x} - \frac{1}{6y}$
23. $\frac{3}{c^3} - \frac{4}{c^4}$
24. $\frac{5}{6a^2 b} + \frac{2}{9ab^2}$
25. \( \frac{3}{x} + \frac{5}{x+2} \)

27. \( 5 + \frac{4}{x-2} \)

29. \( \frac{1}{2r} + \frac{1}{3s} + \frac{1}{4t} \)

31. \( x - \frac{x}{x+7} \)

33. \( \frac{1}{a} + \frac{1}{a^2} + \frac{1}{a^3} \)

35. \( \frac{3}{x-5} + \frac{4}{5-x} \)

37. \( \frac{2}{x^2+2x-15} - \frac{1}{x^2-5x+6} \)

39. \( \frac{1}{x+1} - \frac{2}{(x+1)^2} + \frac{3}{x^2-1} \)

26. \( \frac{1}{x-1} - \frac{1}{x+1} \)

28. \( \frac{3x}{x^2-4} - \frac{1}{x-2} \)

30. \( \frac{2x-1}{3} - \frac{x-4}{5} \)

32. \( \frac{3}{x} + \frac{2}{x-1} - \frac{5}{x^2-x} \)

34. \( \frac{x}{x^3-x^2} + \frac{1}{x^3} \)

36. \( \frac{x}{x^2+x-6} + \frac{x+1}{x^2+7x+12} \)

38. \( \frac{7x-1}{x^2-9x+20} - \frac{1}{x-5} \)

40. \( \frac{t+1}{2t^2+5t-3} + \frac{t+3}{2t^2-3t+1} \)