

2016 John O'Bryan Mathematical Competition
Questions for the Two-Person Speed Event

*****Calculators may not be used on the first four questions*****

1. There are $k < 25$ players in a game who stand in a circle. The players are numbered consecutively beginning with 1. Player 1 stays in. Player 2 is knocked out. Player 3 stays in. Player 4 is knocked out. This process continues, knocking every other player out, until only one player remains. Find the sum of all values of k for which the Player 13 will be the last player remaining.
2. Let k be the smallest integer such that kx **must** be greater than $x + 2$, if $0.6 < x < 1$. All sides of a right triangle have integer length. If 10 is the length of the shortest side, let w be the smallest possible length of the longest side. Find the value of $(k + w)$.
3. Let $a\pi$ be the area of the circumscribed circle of a triangle whose side-lengths are 12, 16, and 20. The three points $(4,5)$, $(-7,17)$, and $(-95,b)$ are collinear. Find the value of $(a + b)$.
4. Functions f and g are defined as follows: $f(x) = \frac{1-2x}{3}$ and $g(x) = \frac{1-3x}{2}$. Find the value of $f(g(2)) + g(f(3)) + f(g(4)) + g(f(5))$

*****Calculators may be used on the remaining questions*****

5. Let k be the number of gallons of a 50% potassium chloride solution that are added to 15 gallons of a 30% potassium chloride solution to produce a 35% potassium chloride solution. Let w be the number of sides of a regular polygon whose degree measure of one of the exterior angles is 8. Find the value of $(k + w)$.
6. Let q be a positive integer less than 200 such that $\sqrt{q} = \sqrt{d} + \sqrt{d} + \sqrt{d}$ where d is a positive integer. Find the sum of all distinct values of q .
7. Let $S = \log\left(\frac{1}{2}\right) + \log\left(\frac{2}{3}\right) + \log\left(\frac{3}{4}\right) + \dots + \log\left(\frac{n}{n+1}\right) + \dots + \log\left(\frac{99}{100}\right)$. The length k of a side of an equilateral triangle is also a root of the quartic equation $k^4 - 474k^2 - 4840 = 0$. Find the value of $(S + k)$.
8. Let g be the number of distinct permutations of the letters in the word "geometry". Let p be the probability of drawing two hearts if two cards are selected (without replacement) at random from four hearts and two spades. Find the value of the product (gp) .
9. (T1) Calculate the value of $\sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \dots}}}}$. Give your answer to four significant figures.
10. (T2) Find the value of $11_{\text{two}} + 222_{\text{three}} + 3333_{\text{four}}$. Express your answer in **base five**.

Names: _____

School: _____

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Note: All answers must be written legibly and in simplest form. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required. Each problem has the same point-value; however ties for individual awards will be broken based on problem difficulty.

SCORE

1. _____

2. _____

3. _____

4. _____

5. _____

6. _____

7. _____

8. _____

T1. _____

T2. _____

Calculators are not allowed to be used on the first four questions!

This competition consists of eight competitive rounds. Correct answers will receive the following scores:

1st: 7 points
2nd: 5 points
All Others: 3 points

There is a three minute time limit on each round. You may submit only one answer each round. To submit your answer, fold this sheet **lengthwise** and hold it high in the air so that a proctor may check your answer.

SCORE

Name: _____ **ANSWERS** _____

Team Code: _____

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1. 36

2. 31

3. 213

4. 14

5. 50

6. 2277

7. 20

8. 8064

T1. 1.618 (exact required)

T2. 2114_{five} (base optional)

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