

2017 John O'Bryan Mathematical Competition
Questions for the Two-Person Speed Event

*****Calculators may not be used on the first four questions*****

1. 120 students were provided a provided a choice of breakfast. They had three choices: fruit, cereal, and/or yogurt. 53 students had cereal, 47 students had yogurt, and 47 students had fruit. Also, 38 students had only cereal, 12 students had yogurt and fruit but no cereal, 27 students had only fruit, and 25 students had only yogurt. Find the number of students that chose not to have breakfast.
2. Given $9x - 4y = 7$, let k be the value of y when $x = -5$. Let w be the value of $p^2 - 11$ when $p = -3$. Find the sum $(k + w)$.
3. Two students make a New Year's resolution to get more exercise. One student decides to go to a health club aerobics class every other day, and the other decides to go every third day. They go together on January 2. *How many other* days in January (31 day month) will they be in aerobics class together?
4. Let $k = 3 + 1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \dots$ and let $w = 1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} - \dots$. Find the sum $(k + w)$.
Express your answer as a common or improper fraction reduced to lowest terms.

*****Calculators may be used on the remaining questions*****

5. Find the absolute difference between the numerical area and the numerical perimeter of a rectangle with once side of length 12 and a diagonal of length 20.
6. In $\triangle ABC$, $A = (1, 5)$, $B = (-2, -3)$ and $C = (5, -7)$. If k represents the slope of the altitude in $\triangle ABC$ from vertex B and w represents the slope of the median in $\triangle ABC$ from vertex B , find the product (kw) . Express your answer as a common or improper fraction reduced to lowest terms.
7. The areas of the three of the faces of a right rectangular solid are numerically 20, 45, and 50. Find the numerical volume of this solid. Express your answer in the form $a\sqrt{b}$.
8. Alyssa flipped a fair coin three times, while Sally flipped another fair coin four times, each flip resulting in a head or a tail. Find the probability that Alyssa and Sally flipped the same number of heads. Express your answer as a common fraction reduced to lowest terms.
9. (Tiebreaker #1) In the State of Confusion, truck license plates are made with a combination of the digits 0-9 and the normal 26 letters of the alphabet. License plate numbers consist of three different digits followed by two letters, except the letters O and I cannot be used (example plate: 834BB). Find the number of possible license plates.
10. (Tiebreaker #2) Calculate $(41)_{five} + (23)_{seven}$. Give your answer in base *nine*.

Name: _____ **ANSWERS** _____

Team Code: _____

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Answers for the Two-Person Speed Event**

Note: All answers must be written legibly and in simplest form. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required. Each problem has the same point-value; however ties for individual awards will be broken based on problem difficulty.

1. **3**

2. **-15**

3. **4**

4. **31/6** (must be reduced fraction)

5. **136**

6. **2/15** (must be reduced fraction)

7. **$150\sqrt{2}$**

8. **35/128** (must be reduced fraction)

T1. **414720**

T2. **42_{nine} or 42**

Calculators are not allowed to be used on the first four questions!

This competition consists of eight competitive rounds. Correct answers will receive the following scores:

1st: 7 points
2nd: 5 points
All Others: 3 points

There is a three minute time limit on each round. You may submit only one answer each round. To submit your answer, fold this sheet **lengthwise** and hold it high in the air so that a proctor may check your answer.